

More Dynamical Supersymmetry Breaking ¹

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Abstract

In this paper we introduce a new class of theories which dynamically break supersymmetry based on the gauge group $SU(n) \times SU(3) \times U(1)$ for even n . These theories are interesting in that no dynamical superpotential is generated in the absence of perturbations. For the example $SU(4) \times SU(3) \times U(1)$ we explicitly demonstrate that all flat directions can be lifted through a renormalizable superpotential and that supersymmetry is dynamically broken. We derive the exact superpotential for this theory, which exhibits new and interesting dynamical phenomena. For example, modifications to classical constraints can be field dependent. We also consider the generalization to $SU(n) \times SU(3) \times U(1)$ models (with even $n > 4$). We present a renormalizable superpotential which lifts all flat directions. Because $SU(3)$ is not confining in the absence of perturbations, the analysis of supersymmetry breaking is very different in these theories from the $n = 4$ example. When the $SU(n)$ gauge group confines, the Yukawa couplings drive the $SU(3)$ theory into a regime with a dynamically generated superpotential. By considering a simplified version of these theories we argue that supersymmetry is probably broken.

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1 Introduction

After a lull of about ten years, the number of known models which dynamically break supersymmetry has been steadily rising. One begins to suspect that the restricted number of theories was primarily due to a limited ability to analyze strongly interacting theories. With recent advances in understanding these theories [1, 2], progress is being made in exploring the larger class of theories which can break supersymmetry, leading to several new models of supersymmetry breaking [3, 4, 5, 6]. A second problem with the search for supersymmetry breaking is that theories with even a slightly complicated field content can quickly become cumbersome to analyze. This second problem can still be a frustration.

In this paper, we present an interesting nontrivial application of exact methods to analyze a model which spontaneously breaks supersymmetry. The theories that we analyze are based on the gauge group $SU(n) \times SU(3) \times U(1)$. Because the gauge dynamics are very different for $n = 4$ and $n > 4$, we first consider the gauge group $SU(4) \times SU(3) \times U(1)$. The particular models we explore in this paper are based on an idea discussed in Ref. [3], where it was suggested to search for models which dynamically break supersymmetry by taking a known model and removing generators to reduce the gauge group. This method is guaranteed to generate an anomaly free chiral theory which has the potential to break supersymmetry. There are several known examples of theories with a suitable superpotential respecting the less restrictive gauge symmetries of the resultant theory, in which supersymmetry is broken without runaway directions. However, there is as yet no proof that this method will necessarily be successful.

The $SU(n + 3)$ theories for even n with an antisymmetric tensor and $n - 1$ antifundamentals are known to break supersymmetry dynamically [7]. In this paper we consider models based on the reduced gauge group $SU(n) \times SU(3) \times U(1)$.

Unlike previous models in the literature, neither of the nonabelian gauge groups generates a dynamical superpotential in the absence of the perturbations added at tree level. Because neither factor generates a dynamical superpotential, there is no limit in which the theory can be analyzed perturbatively. Therefore, we derive the exact superpotential for the $n = 4$ case which we use to show supersymmetry is broken in the strongly interacting theory.

The $SU(4) \times SU(3) \times U(1)$ model is interesting for several reasons. First, the demonstration of supersymmetry breaking involves a subtle interplay between the confining dynamics and the tree-level superpotential of the theory. Second, this model implements the mechanism of [5, 6] without introducing additional singlets or potential runaway directions. Third, we can lift all the flat directions by a renormalizable superpotential. Fourth, none of the gauge groups generates a dynamical superpotential; the fields are kept from the origin solely by a quantum modified constraint.

In addition, the exact superpotential exhibits several novel features. First, fields with quantum numbers corresponding to classically vanishing gauge invariant operators emerge, and play the role of Lagrange multipliers for known constraints. Second, we find that classical constraints can be modified not only by a constant, but by field dependent terms which vanish in the classical limit. Third, fields which are independent in the classical theory satisfy linear constraints in the quantum theory. By explicitly substituting the solution to the equation of motion for these fields, we show that quantum analogs of the classical constraints are still satisfied.

The $SU(n) \times SU(3) \times U(1)$ theories for $n > 4$ are less tractable but nonetheless very interesting. We show that it is possible to introduce Yukawa couplings which lift all classical flat directions. We then consider the low-energy limit of this theory. The $SU(3)$ gauge group without the perturbative superpotential is not confining. However, the $SU(n)$ confined theory in the presence of Yukawa couplings induces masses for sufficiently many flavors that there is a dynamical superpotential associated with both the $SU(3)$ and $SU(n)$ dynamics. This low-energy superpotential depends non-trivially on both the strong dynamical scales of the low-energy theory and the Yukawa couplings of the microscopic theory. We consider this model with and without Yukawa couplings which lift the baryon flat directions. In the first case, the theory is too complicated to solve. The form of the low-energy superpotential permitted by the symmetries is nonetheless quite interesting in that it mixes the perturbative and strong dynamics. In the second case, we can explicitly derive that supersymmetry is broken. In either case, there is a spontaneously broken global $U(1)$ symmetry, so we conclude this theory probably breaks supersymmetry and has no dangerous runaway directions when all required Yukawa couplings are nonvanishing.

The outline of this paper is as follows. We first describe the $SU(4) \times SU(3) \times U(1)$ model classically. In particular, we show that the model has

no classical flat directions. In Section 3, we analyze the quantum mechanical theory in the strongly interacting regime. In Section 4, we show that the model breaks supersymmetry. In Section 5, we discuss generalizations to $SU(n) \times SU(3) \times U(1)$ and conclude in the final section.

2 The Classical $SU(4) \times SU(3) \times U(1)$ Theory

The field content of the model we study is obtained by decomposing the chiral multiplets of an $SU(7)$ theory with the field content consisting of an antisymmetric tensor and three anti-fundamentals into its $SU(4) \times SU(3) \times U(1)$ subgroup. The fields are:

$$A^{\alpha\beta}(6, 1)_6, \bar{Q}_a(1, \bar{3})_{-8}, T^{\alpha a}(4, 3)_{-1}, \bar{F}_{\alpha I}(\bar{4}, 1)_{-3}, \bar{Q}_{ai}(1, \bar{3})_4,$$

where $i, I = 1, 2, 3$ are flavor indices, while Greek letters denote $SU(4)$ indices and Latin ones correspond to $SU(3)$. In this notation $(n, m)_q$ denotes a field that transforms as an n under $SU(4)$, m under $SU(3)$ and has $U(1)$ charge q .

We take the classical superpotential to be

$$W_{cl} = A^{\alpha\beta} \bar{F}_{\alpha 1} \bar{F}_{\beta 2} + T^{\alpha a} \bar{Q}_{a1} \bar{F}_{\alpha 1} + T^{\alpha a} \bar{Q}_{a2} \bar{F}_{\alpha 2} + T^{\alpha a} \bar{Q}_{a3} \bar{F}_{\alpha 3} + \bar{Q}_a \bar{Q}_{b2} \bar{Q}_{c1} \epsilon^{abc}. \quad (1)$$

We will show shortly that this superpotential lifts all D-flat directions.

From the fundamental fields we can construct operators which are invariant under the gauge symmetries of the theory. We first list those which are invariant under $SU(4) \times SU(3)$ and subsequently construct operators which are also $U(1)$ invariant. Later on it will be important to distinguish operators invariant under the confining gauge groups but which carry $U(1)$ charge.

$$\begin{aligned} M_{iI} &= T^{\alpha a} \bar{Q}_{ai} \bar{F}_{\alpha I} & 0 \\ M_{4I} &= T^{\alpha a} \bar{Q}_a \bar{F}_{\alpha I} & -12 \\ X_{IJ} &= A^{\alpha\beta} \bar{F}_{\alpha I} \bar{F}_{\beta J} & 0 \\ X_{I4} &= \frac{1}{6} A^{\beta\alpha} \bar{F}_{\beta I} \epsilon_{\alpha\gamma\delta\zeta} T^{\gamma a} T^{\delta b} T^{\zeta c} \epsilon_{abc} & 0 \\ \text{Pf } A &= \epsilon_{\alpha\beta\gamma\delta} A^{\alpha\beta} A^{\gamma\delta} & 12 \\ Y_{ij} &= \epsilon_{\alpha\beta\gamma\delta} A^{\alpha\beta} T^{\gamma a} \bar{Q}_{ai} T^{\delta b} \bar{Q}_{bj} & 12 \end{aligned} \quad (2)$$

$$\begin{aligned}
Y_{i4} &= \epsilon_{\alpha\beta\gamma\delta} A^{\alpha\beta} T^{\gamma a} \bar{Q}_{ai} T^{\delta b} \bar{Q}_b & 0 \\
\bar{B} &= \frac{1}{6} \bar{F}_{\alpha I} \bar{F}_{\beta J} \bar{F}_{\gamma K} \epsilon^{IJK} T^{\alpha a} T^{\beta b} T^{\gamma c} \epsilon_{abc} & -12 \\
\bar{b}^i &= -\frac{1}{2} \bar{Q}_a \bar{Q}_{bj} \bar{Q}_{ck} \epsilon^{ijk} \epsilon^{abc} & 0 \\
\bar{b}^4 &= \frac{1}{6} \bar{Q}_{ai} \bar{Q}_{bj} \bar{Q}_{ck} \epsilon^{ijk} \epsilon^{abc} & 12
\end{aligned}$$

The right hand side column indicates the charges of the operators under the $U(1)$ gauge group. All other $SU(4) \times SU(3)$ invariants can be obtained as products of these operators. The classical constraints obeyed by these fields are:

$$\begin{aligned}
4 X_{I4} X_{JK} \epsilon^{IJK} - \bar{B} \text{Pf} A &= 0 \\
\epsilon^{ijk} \epsilon^{IJK} (\text{Pf} A M_{iI} M_{jJ} M_{kK} - 6 Y_{ij} M_{kI} X_{JK}) &= 0 \\
\epsilon^{ijk} \epsilon^{IJK} (\text{Pf} A M_{4I} M_{jJ} M_{kK} - 2 Y_{jk} M_{4I} X_{JK} + 4 Y_{j4} M_{kI} X_{JK}) &= 0 \\
Y_{i4} \bar{b}^i &= 0 \\
\bar{B} \bar{b}^4 - \frac{1}{6} \epsilon^{ijk} \epsilon^{IJK} M_{iI} M_{jJ} M_{kK} &= 0 \\
\bar{B} \epsilon^{kij} Y_{ij} - 2 \epsilon^{kij} \epsilon^{IJK} M_{iI} M_{jJ} X_{K4} &= 0 \\
M_{4I} \bar{b}^4 + M_{iI} \bar{b}^i &= 0 \\
\epsilon^{ijk} Y_{jk} M_{4I} + 2 \epsilon^{ijk} M_{jI} Y_{k4} + 4 X_{I4} \bar{b}^i &= 0 \\
\epsilon^{IJK} \epsilon^{ijk} M_{iI} M_{jJ} M_{4K} Y_{k4} &= 0.
\end{aligned} \tag{3}$$

The completely gauge invariant fields can be formed by taking products of the above $U(1)$ charged fields. However, most of these combinations turn out to be products of other completely gauge invariant operators. As an operator basis we can use the neutral fields from Eq. 2 and $E_I = M_{4I} \text{Pf} A$. These operators are subject to the following classical constraints:

$$\begin{aligned}
\epsilon^{IJK} E_J M_{iK} \bar{b}^i &= 0 \\
Y_{i4} \bar{b}^i &= 0 \\
\epsilon^{IJK} \epsilon^{ijk} M_{iI} M_{jJ} E_K Y_{k4} &= 0 \\
\epsilon^{IJK} \epsilon^{ijk} M_{iI} M_{jJ} Y_{k4} M_{lK} \bar{b}^l &= 0
\end{aligned} \tag{4}$$

These constraints follow from Eq. 3. We have omitted the linear constraints following from Eq. 3 which define additional unnecessary fields. These operators obeying the above constraints parameterize the D-flat directions of the theory.

In terms of the invariants defined above we can express the superpotential as

$$W_{cl} = X_{12} + M_{11} + M_{22} + M_{33} + \bar{b}^3. \quad (5)$$

We now show that this superpotential suffices to lift all D -flat directions. It is easiest to show this (using the results of Ref. [8]) by demonstrating that the holomorphic invariants which parameterize the flat directions are all determined by the equations of motion (as opposed to parameterizing the flat directions in terms of the fundamental fields). If all holomorphic invariants are determined, we can conclude that all potential flat directions are lifted.

We consider the equations of motion corresponding to the classical superpotential of Eq. 1. The equation $\frac{\partial W}{\partial A}$ sets X_{12} to zero if we multiply by A . Forming all gauge invariant combinations from $\frac{\partial W}{\partial \bar{Q}_{ai}}$ we obtain the following. Multiplying $\frac{\partial W}{\partial \bar{Q}_{a3}}$ by \bar{Q}_{aj} gives

$$M_{j3} = 0,$$

similarly for $\frac{\partial W}{\partial \bar{Q}_{a1,2}}$ we obtain

$$\begin{aligned} M_{12} = 0 \quad M_{22} + \bar{b}^3 = 0 \quad M_{32} - \bar{b}^2 = 0 \\ M_{21} = 0 \quad M_{11} + \bar{b}^3 = 0 \quad M_{31} - \bar{b}^1 = 0. \end{aligned}$$

Next, we multiply the same equations by $\epsilon_{abc} T^{\beta b} T^{\gamma c} A^{\delta \rho} \epsilon_{\beta \gamma \delta \rho}$ to obtain

$$X_{34} = 0 \quad Y_{24} + 2X_{14} = 0 \quad Y_{14} - 2X_{24} = 0.$$

Also, by multiplying $\frac{\partial W}{\partial \bar{Q}_{ai}}$ by $\bar{Q}_a \text{Pf} A$ we get

$$E_I = 0.$$

Next, from $\frac{\partial W}{\partial \bar{Q}_a} \bar{Q}_a$ we obtain that

$$\bar{b}^3 = 0.$$

We obtain the remaining equations from $\frac{\partial W}{\partial F_{\alpha I}}$. They are:

$$\begin{aligned} M_{13} - X_{23} = 0 \quad M_{23} + X_{13} = 0 \quad M_{3I} = 0 \\ E_2 + 4Y_{14} = 0 \quad E_1 - 4Y_{24} = 0 \quad Y_{34} = 0 \end{aligned}$$

The only solution to these equations sets all operators to be zero. Therefore, our theory does not have flat directions.

In Ref. [9] it was argued that theories which have no flat directions, but preserve an anomaly free R symmetry break supersymmetry spontaneously if the $U(1)_R$ symmetry is spontaneously broken in the vacuum. This follows because there would be a massless pseudoscalar, which is unlikely to have a massless scalar partner. The superpotential of Eq. 1 preserves an R symmetry under which the R charges are $R(A) = R(\bar{F}_3) = 0$, $R(\bar{F}_1) = R(\bar{F}_2) = 1$, $R(\bar{Q}_1) = R(\bar{Q}_2) = \frac{5}{3}$, $R(\bar{Q}_3) = \frac{8}{3}$, $R(\bar{Q}) = -\frac{4}{3}$ and $R(T) = -\frac{2}{3}$. Although this symmetry is anomalous with respect to the $U(1)$ gauge group, if it is spontaneously broken, the associated Goldstone boson is nonetheless massless so the argument of Ref. [9] should still apply.

Notice that the classical equations of motion in our theory have a solution only where all fields vanish. In the next section we show that the quantum theory does not permit such a supersymmetric solution, so that supersymmetry is broken.

3 The Quantum $SU(4) \times SU(3) \times U(1)$ Theory

In this section we will derive the exact superpotential of the $SU(4) \times SU(3) \times U(1)$ theory. The fact that it is possible to determine the exact superpotential of the theory will enable us to prove that supersymmetry is dynamically broken.

Before proceeding, we list the global symmetries of the microscopic fields, which are useful when constraining the form of the exact superpotential. The global symmetries are:

| | $U(1)_A$ | $U(1)_{\bar{Q}}$ | $U(1)_T$ | $U(1)_{\bar{F}}$ | $SU(3)_{\bar{F}_I}$ | $U(1)_{\bar{Q}_i}$ | $SU(3)_{\bar{Q}_i}$ | $U(1)_R$ |
|---------------|----------|------------------|----------|------------------|---------------------|--------------------|---------------------|----------|
| A | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| \bar{Q} | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| T | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| \bar{F}_I | 0 | 0 | 0 | 1 | 3 | 0 | 1 | 0 |
| \bar{Q}_i | 0 | 0 | 0 | 0 | 1 | 1 | 3 | 0 |
| Λ_3^5 | 0 | 1 | 4 | 0 | 1 | 3 | 1 | -2 |
| Λ_4^8 | 2 | 0 | 3 | 3 | 1 | 0 | 1 | 0 |

The only invariants under all global symmetries including $U(1)_R$ are $\mathcal{A} =$

$X_{IJ}X_{K4}\epsilon^{IJK}/\Lambda_4^8$ and $\mathcal{B} = \bar{B}\text{Pf}A/\Lambda_4^8$.

We now identify the proper degrees of freedom. To do so, it is convenient to first take the limit $\Lambda_3 \gg \Lambda_4$ and construct $SU(3)$ invariant operators which are mesons and baryons formed from the $SU(3)$ charged fields, and then to construct the $SU(4)$ bound states of these fields. This gives us the spectrum which matches anomalies of the original microscopic theory, independent of the ratio Λ_3/Λ_4 .

Below the $SU(3)$ scale, the theory can be described by an $SU(4)$ theory with an antisymmetric tensor and four flavors. These four flavors are

$$\begin{aligned}\bar{F}_{\alpha 4} &= \frac{1}{6}\epsilon_{\beta\gamma\delta\alpha}T^{\beta a}T^{\gamma b}T^{\delta c}\epsilon_{abc}, \\ F_i^\alpha &= T^{\alpha a}\bar{Q}_{ai}, \quad i = 1, 2, 3 \\ F_4^\alpha &= T^{\alpha a}\bar{Q}_a,\end{aligned}\tag{6}$$

The three remaining antifundamentals are $\bar{F}_{\alpha I}$, $I = 1, 2, 3$, the original fields. The $SU(3)$ antibaryons are the \bar{b}^i 's of Eq. 2, which are singlets under $SU(4)$.

The four-flavor theory with an antisymmetric tensor has been described in Ref. [10]. The confined states of the $SU(4)$ theory are

$$\begin{aligned}\text{Pf}A &= \epsilon_{\alpha\beta\gamma\delta}A^{\alpha\beta}A^{\gamma\delta} \\ M_{iI} &= F_i^\alpha\bar{F}_{\alpha I} \\ X_{IJ} &= A^{\alpha\beta}\bar{F}_{\alpha I}\bar{F}_{\beta J} \\ Y_{ij} &= A^{\alpha\beta}F_i^\gamma F_j^\delta\epsilon_{\alpha\beta\gamma\delta} \\ B &= \frac{1}{24}F_i^\alpha F_j^\beta F_k^\gamma F_l^\delta\epsilon_{\alpha\beta\gamma\delta}\epsilon^{ijkl} \\ \bar{B} &= \frac{1}{24}\bar{F}_{\alpha I}\bar{F}_{\beta J}\bar{F}_{\gamma K}\bar{F}_{\delta L}\epsilon^{\alpha\beta\gamma\delta}\epsilon^{IJKL}.\end{aligned}\tag{7}$$

Here the indices i and I range from 1 to 4. Note that B, M_{44} and M_{i4} are fields which vanish classically. However, anomaly matching of the microscopic theory to the low-energy theory requires the presence of these fields. Fields other than B, M_{44} and M_{i4} correspond to operators introduced in Eq. 2. The low-energy theory consists of the fields listed in Eq. 2 and the new fields B, M_{44} , and M_{i4} .

In order to construct the superpotential it is again convenient to consider the limit $\Lambda_3 \gg \Lambda_4$. Below the Λ_3 scale, there is an $SU(4)$ theory with four

flavors and an antisymmetric tensor together with the confining $SU(3)$ superpotential of Ref. [1]. The superpotential for the four-flavor $SU(4)$ theory with an antisymmetric tensor has been described in Ref. [10]. We determined the coefficients in the superpotential of Ref. [10] by requiring that the equations of motion reproduce the classical constraints.

In this limit, the superpotential has to be the sum of the contributions from $SU(3)$ and $SU(4)$ dynamics. The exact superpotential is therefore of the form:

$$W = \bar{b}^3 + X_{12} + M_{11} + M_{22} + M_{33} + \frac{1}{\Lambda_3^5} (M_{i4} \bar{b}^i - B) + f(\mathcal{A}, \mathcal{B}) \cdot \frac{1}{24 \Lambda_3^5 \Lambda_4^8} (24 B X_{IJ} X_{KL} \epsilon^{IJKL} + 6 \bar{B} Y_{ij} Y_{kl} \epsilon^{ijkl} - 24 B \bar{B} \text{Pf} A + \text{Pf} A \epsilon^{ijkl} \epsilon^{IJKL} M_{iI} M_{jJ} M_{kK} M_{lL} - 12 \epsilon^{ijkl} Y_{ij} M_{kI} M_{lJ} X_{KL} \epsilon^{IJKL}), \quad (8)$$

where f is an as yet undetermined function of the symmetry invariants \mathcal{A} and \mathcal{B} , and $i, I = 1, \dots, 4$. Therefore, the symmetries together with the limit $\Lambda_3 \gg \Lambda_4$ restrict the superpotential up to a function of \mathcal{A} and \mathcal{B} . However, a negative power series in \mathcal{A} or \mathcal{B} would imply unphysical singularities, since there is no limit in which the number of flavors in the $SU(4)$ theory is less than the number of colors. On the other hand, a positive power series in \mathcal{A} or \mathcal{B} would not correctly reproduce the limit where $\Lambda_4 \gg \Lambda_3$. In this limit one has an $SU(4)$ theory with an antisymmetric tensor and three flavors, which yields a quantum modified constraint [4]. Observe the amazing fact that the B equation of motion which involves the superpotential from both the $SU(3)$ and $SU(4)$ terms exactly reproduces this $SU(4)$ quantum modified constraint. This is only true with no further modification of the second term. In fact, this is what permits us to fix the relative coefficient of the two terms in parentheses. Thus we conclude that $f(\mathcal{A}, \mathcal{B}) \equiv 1$.

We stress again that each of the fields B , M_{i4} , and M_{44} vanish classically. In the quantum theory, the B field acts as a Lagrange multiplier for the three flavor $SU(4)$ quantum modified constraint. The M_{i4} and M_{44} equations of motion are

$$\begin{aligned} \epsilon^{ijk} \epsilon^{IJK} (\text{Pf} A M_{iI} M_{jJ} M_{kK} - 6 Y_{ij} M_{kI} X_{JK}) &= 6 \Lambda_4^8 \bar{b}^4 \\ \epsilon^{ijk} \epsilon^{IJK} (\text{Pf} A M_{4I} M_{jJ} M_{kK} - 2 Y_{jk} M_{4I} X_{JK} + 4 Y_{j4} M_{kI} X_{JK}) &= 2 \Lambda_4^8 \bar{b}^i \end{aligned} \quad (9)$$

The linear equations for \bar{b}^i and \bar{b}^4 can be understood by the fact that they appear as mass terms for M_{44} and M_{i4} . The equations of motion in Eq. 9

can be interpreted as quantum modified constraints of a three flavor $SU(4)$ theory with the scales related through the \bar{b} -dependent masses.

It is a nontrivial check on the superpotential of Eq. 8 that all classical constraints have a quantum analog and vice versa. The quantum modified constraints involving \bar{b}^i and \bar{b}^4 are derived by substituting in the solution to their equation of motion. The quantum modified constraints are:

$$4 X_{I4} X_{JK} \epsilon^{IJK} - \bar{B} \text{Pf} A = \Lambda_4^8 \quad (10)$$

$$\epsilon^{ijk} \epsilon^{IJK} (\text{Pf} A M_{iI} M_{jJ} M_{kK} - 6 Y_{ij} M_{kI} X_{JK}) = 6 \Lambda_4^8 \bar{b}^4 \quad (11)$$

$$\epsilon^{ijk} \epsilon^{IJK} (\text{Pf} A M_{4I} M_{jJ} M_{kK} - 2 Y_{jk} M_{4I} X_{JK} + 4 Y_{j4} M_{kI} X_{JK}) = 2 \Lambda_4^8 \bar{b}^i \quad (12)$$

$$\epsilon^{IJK} \epsilon^{ijk} M_{iI} M_{jJ} M_{4K} Y_{k4} = 2 B M_{4I} X_{JK} \epsilon^{IJK} \quad (13)$$

$$\bar{B} \epsilon^{kij} Y_{ij} - 2 \epsilon^{kij} \epsilon^{IJK} M_{iI} M_{jJ} X_{K4} = -2 M_{i4} M_{jI} \epsilon^{kij} X_{JK} \epsilon^{IJK} \quad (14)$$

while the remaining constraints are not modified. The interesting thing to observe in the above equations is that the quantum modifications do not simply involve addition of a constant to the classical field equations. The quantum modification can be field dependent. The classical limit is recovered in Eqs. 13, 14 because B and M_{i4} are fields which vanish classically. Without a tree-level superpotential M_{i4} is set to zero by the \bar{b}^i equations of motion. However, M_{i4} can be non-vanishing in the presence of a tree-level superpotential. The quantum modifications in Eqs. 11, 12 do not contain classically vanishing fields, but are proportional to Λ_4 , which ensures the correct classical limit. This field dependent modification of constraints is a new feature which is not present when analyzing simple nonabelian gauge groups.

Note that five of our constraints (Eqs. 10, 11 and 12) can be interpreted as the quantum modified constraints on the moduli space of an $SU(4)$ gauge theory with an antisymmetric tensor and three flavors. Such a theory is obtained in several limits. If $\Lambda_4 \gg \Lambda_3$ one trivially has a three flavor $SU(4)$ theory with an antisymmetric tensor. On the other hand, if $\Lambda_3 \gg \Lambda_4$ and any single \bar{b} is non-vanishing one also has a three flavor $SU(4)$ theory with its corresponding quantum modified constraint.

When deriving the constraints in Eqs. 10-14 from the exact superpotential we frequently encounter expressions containing inverse powers of Λ_4 . Such terms are singular in the limit when Λ_3 is held fixed and $\Lambda_4 \rightarrow 0$. This is true even for expressions containing the fields B , M_{i4} and M_{44} , since they vanish only in the limit when $\Lambda_3 \rightarrow 0$. Therefore all such terms must and do cancel.

4 Dynamical Supersymmetry Breaking

In the low-energy description of our model the $SU(4)$ and $SU(3)$ gauge groups are confined and the only remaining gauge group is the $U(1)$. This $U(1)$ does not play any role in supersymmetry breaking; its purpose is to lift some classical flat directions. Unlike previous examples of dynamical supersymmetry breaking, the superpotential can be completely analyzed in a regime where there are no singularities, either due to a dynamically generated superpotential present in the initial theory, integrating out fields, or particular limits. If the theory breaks supersymmetry, it is simply of O’Raifeartaigh type [11]. In this section, we show that this is the case; there is no consistent solution of the F -flatness equations for the exact superpotential of Eq. 8.

We first assume that $\bar{B} \neq 0$. Then the $\frac{\partial W}{\partial Y_{ij}}$ equation of motion implies

$$Y_{ij} = \frac{1}{\bar{B}} X_{KL} M_{iI} M_{jJ} \epsilon^{IJKL}. \quad (15)$$

Plugging this expression into the $\frac{\partial W}{\partial X_{IJ}}$ equation of motion, we obtain

$$(\delta_S^3 \delta_T^4 - \delta_T^3 \delta_S^4) + \frac{8}{\Lambda_3^5 \Lambda_4^8} B X_{ST} - \frac{2}{\Lambda_3^5 \Lambda_4^8} \frac{1}{\bar{B}} \epsilon^{ijkl} M_{iM} M_{jN} M_{kS} M_{lT} X_{KL} \epsilon^{MNKL} = 0.$$

However, by using the $\frac{\partial W}{\partial \text{Pf}A} = 0$ equation in the above expression we arrive at a contradiction.

Next we assume that $\bar{B} = 0$, but $B \neq 0$. We can now solve for X using the equation $\frac{\partial W}{\partial X_{IJ}} = 0$:

$$X_{MN} = \frac{\Lambda_3^5 \Lambda_4^8}{8B} \left[(\delta_M^3 \delta_N^4 - \delta_N^3 \delta_M^4) + 48 \epsilon^{ijkl} Y_{ij} M_{kM} M_{lN} \right]. \quad (16)$$

Then we multiply this equation by $\epsilon^{ijkl} \epsilon^{IJMN} M_{kI} M_{lJ}$. The Y_{ij} equation of motion sets the left hand side to zero, while the PfA equation of motion sets the second term on the right hand side to zero. Therefore,

$$\epsilon^{ijkl} M_{iI} M_{jJ} \epsilon^{IJ34} = 0.$$

Using this fact, the PfA equation of motion, and the expression for X_{MN} in Eq. 16 we get that $\frac{\partial W}{\partial B} = -\frac{1}{\Lambda_3^5}$, which again means that the equations of motion are contradictory.

Finally we assume that $B = \bar{B} = 0$. Then the $\frac{\partial W}{\partial X_{IJ}}$ equation of motion implies

$$\epsilon^{ijkl} Y_{ij} M_{kI} M_{lJ} = 0$$

for all I, J except $I = 3, J = 4$. Multiplying the $\frac{\partial W}{\partial X_{IJ}}$ equation of motion by $M_{iI} M_{jJ}$ and using the $\frac{\partial W}{\partial \text{Pf} A}$ equation of motion we get that

$$\epsilon^{ijkl} M_{i1} M_{j2} = 0.$$

Using these results the $\frac{\partial W}{\partial M_{i3}}$ equation of motion yields

$$\delta^{i3} - \frac{1}{\Lambda_3^5 \Lambda_4^8} \epsilon^{ijkl} Y_{jk} M_{lJ} X_{KL} \epsilon^{3JKL} = 0.$$

Multiplying this equation by M_{i4} implies $M_{34} = 0$, which is in contradiction with the $\frac{\partial W}{\partial b^3}$ equation of motion. Thus we have shown that this $SU(4) \times SU(3) \times U(1)$ model breaks supersymmetry dynamically. Since there are no classical flat directions, there should not be runaway directions in this model.

Having presented a general proof of supersymmetry breaking, we now give a simpler proof that applies only in a restricted region of parameter space. Assume that Λ_3 is the largest parameter in the theory. The effective superpotential just below the Λ_3 scale is

$$\begin{aligned} W = & \bar{b}^3 + \gamma A^{\alpha\beta} \bar{F}_{\alpha 1} \bar{F}_{\beta 2} + \lambda_1 F_1^\alpha \bar{F}_{\alpha 1} + \lambda_2 F_2^\alpha \bar{F}_{\alpha 2} + \lambda_3 F_3^\alpha \bar{F}_{\alpha 3} + \\ & \frac{1}{\Lambda_3^5} \left(\bar{F}_{\alpha 4} F_i^\alpha \bar{b}^i - \det F_i^\alpha \right), \end{aligned} \quad (17)$$

where we use the notation from Eq. 6 and we introduced explicitly the Yukawa couplings γ and $\lambda_{1,2,3}$. In terms of the canonically normalized fields, $\lambda_{1,2,3}$ are mass parameters.

Next, we integrate out three of the four flavors to arrive at an $SU(4)$ theory with one flavor and a superpotential

$$W = \bar{b}^3 + \frac{1}{\Lambda_3^5} \bar{F}_{\alpha 4} F_4^\alpha \bar{b}^4. \quad (18)$$

To describe the dynamics of the one-flavor $SU(4)$ theory, it is useful to define the effective one-flavor $SU(4)$ scale $\tilde{\Lambda}_4^5$, which is proportional to $\lambda_1 \lambda_2 \lambda_3 \Lambda_3^5 \Lambda_4^8$.

Below the effective $\tilde{\Lambda}_4$ scale there is a dynamically generated term, so the low-energy superpotential is

$$W = \bar{b}^3 + \frac{1}{\Lambda_3^5} M_{44} \bar{b}^4 + \left(\frac{\tilde{\Lambda}_4^5}{\text{Pf} A M_{44}} \right)^{\frac{1}{2}}, \quad (19)$$

where $M_{44} = \bar{F}_{\alpha 4} F_4^\alpha$. There are no solutions to the equations of motion. Note that the potential runaway direction is removed by the $U(1)$ D-flatness condition. Therefore supersymmetry is dynamically broken. Observe that supersymmetry breaking in this limit has two sources. First the superpotential generated by the $SU(3)$ and $SU(4)$ gauge groups together does not have a supersymmetric minimum. Second, a Yukawa term in the tree level superpotential is confined into a single field which is also a source of supersymmetry breaking. In fact, the tree-level Yukawa terms have three different important roles in this analysis. They lift the flat directions, they yield mass terms for the $SU(4)$ fields after $SU(3)$ is confining, and they also contribute to supersymmetry breaking by the linear term. The fact that there is a quantum modified constraint in the $\Lambda_4 \gg \Lambda_3$ limit of the theory does not seem to play a major role in the dynamics of supersymmetry breaking.

By symmetries, it can be shown that this simpler proof neglects power corrections proportional to

$$\left(\frac{\gamma^2 \bar{b}^i \text{Pf} A M_{44}}{\lambda^4 (\Lambda_3^5)^2} \right)^k.$$

This reflects the fact that here we are studying the effective theory treating Λ_3 as large. The \bar{b}^4 equation of motion together with the fact that there are no flat directions imply broken supersymmetry even with these corrections incorporated.

5 $SU(n) \times SU(3) \times U(1)$ Theories

In this section we generalize the $SU(4) \times SU(3) \times U(1)$ model to $SU(n) \times SU(3) \times U(1)$, with n even. There are several interesting features of the dynamics of these theories. Without a tree-level superpotential the $SU(3)$ group is not confining. However, the Yukawa couplings of the tree-level superpotential become mass terms when the $SU(n)$ group confines. These mass

terms drive the $SU(3)$ group into the confining regime as well. Confinement can change chiral theories into non-chiral ones. In this example Yukawa couplings become mass terms. In fact, the quantum modified constraint associated with the $SU(n)$ group of the initial theory does not appear to play an essential role in the dynamics of supersymmetry breaking. Another interesting phenomena is that even if we remove some of the couplings from the superpotential, so that some flat directions are not lifted, these directions turn out to be lifted in the quantum theory. In particular, once the Yukawa couplings turn into mass terms, the $SU(3)$ antibaryon directions are automatically lifted.

As in Section 2, we obtain the field content for these models by decomposing the fields of the $SU(n+3)$ theory with an antisymmetric tensor and $n-1$ anti-fundamentals to $SU(n) \times SU(3) \times U(1)$:

$$\begin{aligned} \square &\rightarrow A^{\alpha\beta}(\square, 1)_6 + \bar{Q}_a(1, \bar{3})_{-2n} + T^{\alpha a}(\square, 3)_{3-n} \\ (n-1)\bar{\square} &\rightarrow \bar{F}_{\alpha I}(\bar{\square}, 1)_{-3} + \bar{Q}_{ai}(1, \bar{3})_n, \end{aligned} \quad (20)$$

where $i, I = 1, \dots, n-1$.

In analogy to the 4-3-1 case, $SU(n) \times SU(3) \times U(1)$ invariants are:

$$\begin{aligned} M_{iI} &= T^{\alpha a} \bar{Q}_{ai} \bar{F}_{\alpha I} \\ X_{IJ} &= A^{\alpha\beta} \bar{F}_{\alpha I} \bar{F}_{\beta J} \\ X_I &= \frac{1}{6} A^{\alpha_n \alpha_{n-1}} \dots A^{\alpha_4 \beta} \bar{F}_{\beta I} \epsilon_{\alpha_n \dots \alpha_1} T^{\alpha_3 a} T^{\alpha_2 b} T^{\alpha_1 c} \epsilon_{abc} \\ Y_i &= A^{\alpha_n \alpha_{n-1}} \dots A^{\alpha_4 \alpha_3} T^{\alpha_2 a} \bar{Q}_{ai} T^{\alpha_1 b} \bar{Q}_b \\ \bar{b}_{ij} &= \bar{Q}_a \bar{Q}_{bi} \bar{Q}_{cj} \epsilon^{abc} \\ E_I &= \epsilon_{\alpha_n \dots \alpha_1} A^{\alpha_n \alpha_{n-1}} \dots A^{\alpha_2 \alpha_1} T^{\beta a} \bar{Q}_a \bar{F}_{\beta I} \end{aligned} \quad (21)$$

We consider the following superpotential:

$$\begin{aligned} W &= X_{12} + X_{34} + \dots + X_{n-3, n-2} + \bar{b}_{23} + \bar{b}_{45} + \dots + \bar{b}_{n-2, 1} + \\ &\quad M_{11} + M_{22} + \dots + M_{n-1, n-1}. \end{aligned} \quad (22)$$

Observe the relative shifts in the indices between the X and \bar{b} operators. One can check that not all flat directions are removed without such a shift in the indices.

To demonstrate that all flat directions are lifted, one can use the same method as described in Section 2. In this example, we require looking not

only at linear equations in the flat direction fields, but also higher order equations, in order to demonstrate that no flat directions remain in the presence of the tree-level superpotential above.

We first use the \bar{Q}_i and \bar{F}_i equations of motion (contracted with \bar{Q}_k and \bar{F}_j). One will then find potential flat directions which are labeled by $i = 1, 3, 5, \dots, 2[n/4] - 1$ with equal values of $X_{2j-1, (2j-1+i) \parallel (n-2)} = \bar{b}_{2j, (2j+i) \parallel (n-2)}$, where $j = 1, 2, 3, \dots, (n-2)/2$ labels nonvanishing X and \bar{b} fields which are equal along the flat direction. Here, by $[x]$ we denote the greatest integer less than x , while we define $m \parallel n \equiv 1 + (m-1) \text{ Mod } n$. There is another set of potential flat directions of the form $X_{2j, (2j+i) \parallel (n-2)} = \bar{b}_{2j-1, (2j-1+i) \parallel (n-2)}$, where again $j = 1, 2, 3, \dots, (n-2)/2$ and $i = 1, 3, 5, \dots, 2[n/4] - 1$. In the case when $n = 4k$ and $i = k$, two potential flat directions described above are equal to each other, so they represent just one flat direction. Altogether, there are $(n-2)/2$ potential flat directions. One of these flat directions is lifted trivially by the A equation of motion. To see that the remaining flat directions are lifted requires obtaining quadratic equations in the flat direction of fields by suitably contracting the T equations of motion. These equations can be shown to have only the trivial solution where all fields vanish. We have verified this explicitly in the cases $n = 6, 8, 10$, and 12 , but we expect this method to generalize.

One can also verify that the superpotential above preserves two $U(1)$ symmetries, one of which is an R symmetry which is anomalous only with respect to the $U(1)$ gauge group. From the quantum modified constraint it can be shown that at least one of these $U(1)$ symmetries is spontaneously broken. Since the theory has no flat directions and spontaneously breaks a $U(1)$ symmetry, we expect that supersymmetry is broken.

There is a possibility however that in the strongly interacting regime there is a point at which supersymmetry is restored. We now consider the quantum theory and argue that it is likely that supersymmetry is broken.

Without a tree-level superpotential the $SU(3)$ group is not confining for $n > 4$ since $N_f > \frac{3}{2}N_c$. We choose to use fields transforming under $SU(3)$ instead of the $SU(3)$ invariant operators. The D-flatness conditions can then be imposed explicitly. Although in principle one could use holomorphic invariants to parameterize the D-flat directions, the naive application of this method would lead to incorrect results at points of the moduli space where these invariants vanish [12]. Although with careful choice of holomorphic invariants this problem can be circumvented, in practice it is simpler to use

the charged fields when the gauge group is not confining.

The $SU(n)$ group has three flavors and an antisymmetric tensor. Therefore $SU(n)$ is confining and gives rise to a quantum modified constraint as described in Ref. [4]. The $SU(n)$ invariants are:

$$\begin{aligned}
X_{IJ} &= A^{\alpha\beta} \bar{F}_{\alpha I} \bar{F}_{\beta J} \\
m_I^a &= T^{\alpha a} \bar{F}_{\alpha I} \\
\text{Pf} A &= \epsilon_{\alpha_n \dots \alpha_1} A^{\alpha_n \alpha_{n-1}} \dots A^{\alpha_2 \alpha_1} \\
y_a &= A^{\alpha_n \alpha_{n-1}} \dots A^{\alpha_4 \alpha_3} \epsilon_{\alpha_n \dots \alpha_1} T^{\alpha_2 b} T^{\alpha_1 c} \epsilon_{abc}
\end{aligned} \tag{23}$$

together with the fields \bar{Q}_a and \bar{Q}_{ai} .

The superpotential below the Λ_n scale is

$$\begin{aligned}
W &= \alpha^{12} X_{12} + \dots + \alpha^{n-3, n-2} X_{n-3, n-2} + \beta^{23} \bar{Q}_a \bar{Q}_{b2} \bar{Q}_{c3} \epsilon^{abc} + \dots + \\
&\quad \beta^{n-2, 1} \bar{Q}_a \bar{Q}_{b, n-2} \bar{Q}_{c1} \epsilon^{abc} + \lambda^{11} m_1^a \bar{Q}_{a1} + \dots + \lambda^{n-1, n-1} m_{n-1}^a \bar{Q}_{a, n-1} + \\
&\quad \eta \left(\frac{n-2}{3n} \epsilon_{abc} m_{I_1}^a m_{I_2}^b m_{I_3}^c X_{I_4 I_5} \dots X_{I_{n-2} I_{n-1}} \epsilon^{I_1 \dots I_{n-1}} \text{Pf} A - \right. \\
&\quad \left. y_a m_{I_1}^a X_{I_2 I_3} \dots X_{I_{n-2} I_{n-1}} \epsilon^{I_1 \dots I_{n-1}} + \Lambda_n^{2n} \right),
\end{aligned} \tag{24}$$

where η is a Lagrange multiplier and we have explicitly included the coupling constants in the tree-level superpotential. In terms of $SU(n)$ invariants, some of the terms in the above superpotential are just mass terms for $(n-1)$ flavors of $SU(3)$, which drive $SU(3)$ into the confining phase. In the presence of these perturbations, nonperturbative $SU(3)$ dynamics will generate a superpotential. Similar results are found in Ref. [13]. We stress again that in the underlying theory these interactions are Yukawa couplings and not mass terms.

To analyze the low-energy theory, we introduce an additional flavor of $SU(n)$ with mass μ . We do this because the $SU(n)$ quantum modified constraint or equivalently anomaly matching shows that $SU(3)$ must be broken below the scale Λ_n in the original theory. With an additional flavor, the origin of moduli space is permitted and $SU(3)$ can remain unbroken. This permits us to derive the confining superpotential with two massless $SU(3)$ flavors. Although the correct theory is only recovered in the limit $\mu \rightarrow \infty$, we will analyze the theory in the regime $\mu < \Lambda_n$ and hope one can extrapolate the conclusion that supersymmetry is broken [14].

The superpotential with the additional massive $SU(n)$ flavor is:

$$\begin{aligned}
W = & \alpha^{12} X_{12} + \dots + \alpha^{n-3, n-2} X_{n-3, n-2} + \\
& \beta^{23} \bar{Q}_a \bar{Q}_{b2} \bar{Q}_{c3} \epsilon^{abc} + \dots + \beta^{n-2, 1} \bar{Q}_a \bar{Q}_{b, n-2} \bar{Q}_{c1} \epsilon^{abc} + \\
& \lambda^{11} m_1^a \bar{Q}_{a1} + \dots + \lambda^{n-1, n-1} m_{n-1}^a \bar{Q}_{a, n-1} + \mu m_n^4 + \\
& \frac{1}{\Lambda_n^{2n-1}} \left(\text{Pf} A m_{I_1}^a m_{I_2}^b m_{I_3}^c m_{I_4}^d X_{I_5 I_6} \dots X_{I_{n-1} I_n} \epsilon_{abcd} \epsilon^{I_1 \dots I_n} + \right. \\
& Y^{ab} m_{I_1}^c m_{I_2}^d X_{I_3 I_4} \dots X_{I_{n-1} I_n} \epsilon_{abcd} \epsilon^{I_1 \dots I_n} + B X_{I_1 I_2} \dots X_{I_{n-1} I_n} \epsilon^{I_1 \dots I_n} + \\
& \left. \bar{B} Y^{ab} Y^{cd} \epsilon_{abcd} + B \bar{B} \text{Pf} A \right), \tag{25}
\end{aligned}$$

where the variables are as defined in Eq. 23 with an extra $SU(n)$ flavor and

$$\begin{aligned}
B &= T^{\alpha_1 a} T^{\alpha_2 b} T^{\alpha_3 c} F^{\alpha_4 4} A^{\alpha_5 \alpha_6} \dots A^{\alpha_{n-1} \alpha_n} \epsilon_{abc} \epsilon_{\alpha_1 \dots \alpha_n} \\
\bar{B} &= \bar{F}_{\alpha_1 I_1} \dots \bar{F}_{\alpha_n I_n} \epsilon^{I_1 \dots I_n} \epsilon_{\alpha_1 \dots \alpha_n} \\
Y^{a4} &= T^{\alpha_1 a} F^{\alpha_2 4} A^{\alpha_3 \alpha_4} \dots A^{\alpha_{n-1} \alpha_n} \epsilon_{\alpha_1 \dots \alpha_n} \\
Y^{ab} &= \epsilon^{abc} y_c. \tag{26}
\end{aligned}$$

The extra $SU(n)$ flavor is denoted by $F^{\alpha 4}$ and $\bar{F}_{\alpha n}$, and Λ_n is the dynamical scale of the four-flavor $SU(n)$ theory. Here we have not bothered to establish the correct coefficients in the last term in parentheses, since they are irrelevant in the forthcoming analysis.

To arrive at the true low-energy theory, one would integrate out $n - 3$ flavors, at which point a superpotential is generated involving Λ_3 for the four flavor theory. Upon integrating out the two remaining heavy flavors, one would generate a complicated superpotential, involving both the Yukawa couplings and the dynamical scales Λ_n and Λ_3 . It is however technically difficult to explicitly perform this procedure because of the nonlinear terms induced by the baryon operators in the tree-level superpotential.

If we instead constrain the form of the low-energy superpotential with symmetries and limits, we find that the analysis remains quite complicated, because many terms are permitted by the symmetries and physical limits. We deduce the allowed terms by introducing a parameter $\tilde{\Lambda}_3$ which transforms under anomalous global symmetries associated with the rotation of each field carrying $SU(3)$ gauge charge in the initial microscopic theory. Alternatively, we can define Λ_3 for the two flavor theory, where all heavy flavors have been integrated out. The parameters $\tilde{\Lambda}_3^{9-n} \det(\lambda^{iI}) / \Lambda_n^{2n-1}$ and Λ_3^7 have the

same charge under all anomalous symmetries so we can describe the low energy dynamics in terms of either one. We also see that if we consider $\tilde{\Lambda}_3$ as a fundamental finite parameter of the initial theory, singularities in the Yukawa couplings λ^{iI} are permitted when we express the result in terms of the low-energy Λ_3 , since the appropriate ratio is finite. In essence, the Yukawa couplings become mass terms in the $SU(n)$ confined theory, and appear in the matching of Λ_3 across mass thresholds.

Examples of terms permitted by all symmetries and limits are:

$$\frac{\Lambda_3^7}{\Lambda_n^{2n-1}} \frac{\beta^{ij}}{(\lambda^{iI})^2} (X_{IJ})^{(n-4)/2} \text{Pf} A M_I^4 \frac{1}{y_a Y^{a4}},$$

$$\frac{\Lambda_3^{14}}{\Lambda_n^{2n-1}} \frac{(\beta^{ij})^2}{(\lambda^{iI})^4} X_{In} (X_{IJ})^{(n-6)/2} \text{Pf} A \frac{1}{(y_a Y^{a4})(y_a M_n^a)},$$

where β^{ij} 's are the coefficients of the baryon operators $\bar{Q}\bar{Q}_i\bar{Q}_j$, and λ^{iI} of the $T\bar{F}_I\bar{Q}_i$ terms in the tree-level superpotential, but the index structure is not specified. These terms mix the effects of the strong dynamics with the tree-level superpotential, which is purely a consequence of integrating out heavy fields. This does not violate the conjecture of Refs. [1, 15], which states that the couplings of the light fields are not mixed into the dynamically generated superpotential.

Because of the complicated superpotential, the analysis of the full theory is difficult. We will therefore consider a simpler version of the theory, in which the baryon couplings, β^{ij} , are zero. This simplified superpotential does not lift all flat directions classically, which might lead to runaway directions in the quantum theory. One can show that these remaining classical flat directions can be parameterized by the baryon operators \bar{b}_{ij} . However, in the $SU(n)$ confined theory, these fields are not flat, since the terms proportional to m_{iI} , which are Yukawa couplings in the classical theory, are mass terms in the confined theory. In this case, there is a potential for the baryon fields which drives them towards the origin, and the baryon flat directions are lifted in the quantum theory. This is similar in spirit to what was found in Ref. [5]. In that example however, a quadratic constraint becomes a linear constraint so the flat direction is removed; here we simply see that the $SU(n)$ confined superpotential is such that the baryon fields are not flat. However there is a caveat to this analysis which we discuss shortly.

In this limit it is simple to integrate out the heavy flavors and arrive at

the low-energy theory. The resulting superpotential is

$$\begin{aligned}
W = & \frac{1}{\Lambda_n^{2n-1}} \left(y_a m_n^a m_{I_1}^4 X_{I_2 I_3} \dots X_{I_{n-2} I_{n-1}} \epsilon^{I_1 \dots I_{n-1}} + B X_{I_1 I_2} \dots X_{I_{n-1} I_n} \epsilon^{I_1 \dots I_n} \right. \\
& + \bar{B} Y^{a4} y_a + B \bar{B} \text{Pf} A \left. \right) + \mu m_n^4 + \lambda^{12} X_{12} + \dots + \lambda^{n-3, n-2} X_{n-3, n-2} \\
& + \frac{\Lambda_3^7}{(Y^{a4} y_a)(m_n^b \bar{Q}_b) - (Y^{a4} \bar{Q}_a)(m_n^b y_b)}. \tag{27}
\end{aligned}$$

This superpotential clearly breaks supersymmetry since m_n^4 appears only in the term μm_n^4 . Since the scales of the $SU(n)$ theory with and without extra flavor are related by $\mu \Lambda_n^{2n-1} = \Lambda_n^{2n}$, this presumably implies that supersymmetry breaking is characterized by Λ_n^{2n-1} in the original theory.

Thus we just showed that if the $SU(n)$ gauge group is confining, supersymmetry is broken. Had supersymmetry not been broken, this would have been a good assumption, since all operators involving fields transforming under the $SU(n)$ are driven to the origin by the classical potential. Because supersymmetry is broken, it is conceivable that the true vacuum is in the Higgs, rather than the confining phase. Nonetheless, we still expect supersymmetry to be broken since there are no classically flat directions in the theory. In this case however, the \bar{b} operators are not lifted by the superpotential. Once the effect of supersymmetry breaking and the Kähler potential are included, the \bar{b} fields presumably have a nontrivial potential. We have not analyzed whether or not this can give rise to runaway directions, should the Higgs phase prove to be the true vacuum.

Having argued that supersymmetry is probably broken for $\beta^{ij} = 0$, we hope that by including the remaining couplings, while lifting the flat directions, does not introduce a supersymmetric minimum. We expect that the arguments presented above indicate that supersymmetry is broken in the full $SU(n) \times SU(3) \times U(1)$ theories.

6 Conclusions

We have explored a new class of theories based on a product group, in which neither gauge group generates a dynamical superpotential in the absence of perturbations. Nonetheless by exploring the exact superpotential, we could explicitly demonstrate that supersymmetry is broken in the $SU(4) \times SU(3) \times$

$U(1)$ model. We also found interesting phenomena in the exact superpotential, which were discussed in Section 3. For the $SU(n) \times SU(3) \times U(1)$ models, we have found that the exact superpotential is quite complicated. However, in theories with $\beta^{ij} = 0$, we could demonstrate supersymmetry breaking with the addition of an extra flavor of $SU(n)$. In this theory, we also found a large number of classically flat directions which are lifted in the quantum mechanical theory. This is due to the fact that when $SU(n)$ confines, some of the Yukawa couplings in the tree-level superpotential turn into mass terms. This drives the $SU(3)$ group into the confining region and also lifts some of the classical flat directions. Although the particular example we studied in this paper involved a gauge group which had a quantum modified constraint, this fact does not seem essential to supersymmetry breaking in the $SU(n) \times SU(3) \times U(1)$ models, and the same mechanism should apply more generally.

That such interesting features appear in a fairly straightforward example seems indicative of future possibilities. Although the classical theory is constructed according to “standard” rules, in that one can lift all flat directions and spontaneously break an R symmetry, the breaking of supersymmetry is more subtle than in previous models. Verifying that supersymmetry is broken in the full strongly interacting theory is complicated because of the presence of many fields, even when the strong dynamics is well understood. It might be thought that the above properties are sufficient for supersymmetry breaking; however it is not clear to us that there cannot exist a point in the strongly interacting theory at which supersymmetry is preserved. Ultimately it would be interesting if it can more rigorously be shown that models with the above properties necessarily break supersymmetry.

Another intriguing observation is that the theories based on an existing supersymmetric theory with generators removed from the original gauge group with a sufficiently general superpotential seem to permit supersymmetry breaking with no dangerous flat directions. In this paper, we have explored an example distinct from previous ones in which the subgroup of the initial gauge group is a product group for which neither group generates a dynamical superpotential. We have shown that supersymmetry is broken in this case as well, and presumably many other examples can be constructed along these lines and analyzed with the full power of recent developments in strongly interacting gauge theories. It would be worthwhile to analyze these theories, and also to see whether it can be proven in general that theo-

ries constructed in this fashion with a sufficiently general superpotential will break supersymmetry without runaway directions.

We have not addressed the issue of the applicability of our models to visible sector scenarios. In the $SU(4) \times SU(3) \times U(1)$ model, the original theory can preserve a global $SU(2)$ symmetry, and the $SU(n) \times SU(3) \times U(1)$ model preserves a global $U(1)$ (in addition to the R symmetry). Since we have not analyzed the vacuum of our theories in detail, we have not checked whether any of the global symmetries of the classical theory were preserved by the supersymmetry breaking vacuum. The $SU(n) \times SU(3) \times U(1)$ theories with $\beta^{ij} = 0$ perhaps suggest interesting possibilities, since there are many fields which seem to play no role in supersymmetry breaking. There is a possibility that gauge and/or global symmetries in this or similar models are left unbroken. It might be possible to allow for more direct couplings between the supersymmetry and visible sectors in this case.

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